

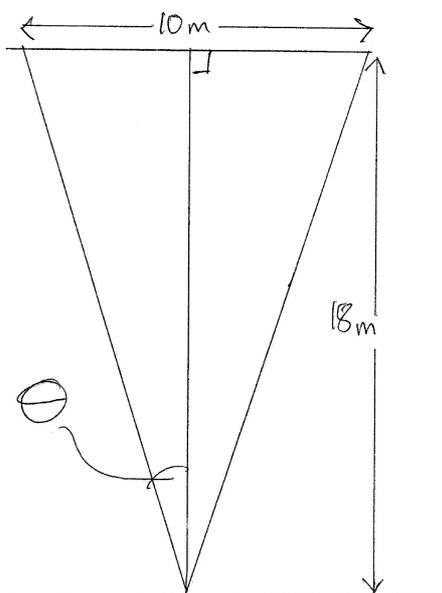
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Answers

Practice questions

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1 Assume one lamp is placed at the nose and the other half way along the underside and the two beams form an isosceles triangle, height 18 m, base 10m (about half the length of the plane).



$$\tan \theta = 5 \text{ m} / 18 \text{ m} = 0.278$$

$$\theta = 16^\circ \text{ (rounded to the nearest degree as this is an estimate)}$$

If you made a different assumptions about the separation of the lamps and the way they were angled (e.g. if you assumed that one lamp was pointing vertically downwards) your answer will be different.

2 Assuming negligible air resistance, the horizontal position of the projectile x is described by the equation

$$x = u_h t$$

The projectile is accelerating vertically with an acceleration g . The vertical position of the projectile, y , is described by the equation

$$y = h - \frac{1}{2} g t^2$$

Substituting for t from the first equation gives

$$y = h - \frac{1}{2} g \left(\frac{x}{u_h} \right)^2$$

which has the same form as

$$y = h - cx^2$$

The constant c depends on the acceleration due to gravity and the initial horizontal velocity of the projectile.

Box 1

Volume of explosive

Use data from Table 1.1 to calculate the density of the Torpex explosive, ρ_E .

Chemical	% composition	Density / kg m^{-3}
RDX	42	1820
TNT	40	1650
Powdered aluminium	18	705

$$\begin{aligned}\rho_E &= 0.42 \times 1820 \text{ kg m}^{-3} + 0.40 \times 1650 \text{ kg m}^{-3} + 0.18 \times 705 \text{ kg m}^{-3} \\ &= 1551.3 \text{ kg m}^{-3}\end{aligned}\quad (1.1)$$

Radius of explosive

The mass, M_E , of explosive is $2.7 \times 10^3 \text{ kg}$.

The volume, V_E , of the explosive is therefore

$$V = \frac{M}{\rho} = \frac{2.7 \times 10^3 \text{ kg}}{1550 \text{ kg m}^{-3}} = 1.742 \text{ m}^3 \quad (1.2)$$

The explosive occupies cylinder of radius r and length l :

$$V = \pi r^2 l \quad (1.3)$$

$$l = 1.47 \text{ m} \approx 1.5 \text{ m}$$

so

$$\begin{aligned}r &= (V/\pi l)^{1/2} \\ &= (1.742 \text{ m}^3/\pi \times 1.5 \text{ m})^{1/2} \\ &\approx 0.6 \text{ m}\end{aligned}\quad (1.4)$$

Volume and mass of steel shell

The surface area, A , of the cylindrical bomb, length l and radius r , is

$$\begin{aligned}A &= 2\pi r l + 2\pi r^2 = 2\pi r(l + r) \\ &= 2\pi \times 0.6 \text{ m} \times (1.5 \text{ m} + 0.6 \text{ m}) \\ &= 7.9 \text{ m}^2\end{aligned}\quad (1.5)$$

The volume V_s of the shell, thickness t (2.54 cm), is

$$\begin{aligned}V_s &= At \\ &= 7.9 \text{ m}^2 \times 0.0254 \text{ m}\end{aligned}\quad (1.6)$$

$$= 0.20 \text{ m}^3$$

The mass M_s of steel, density ρ_s (7900 kg m^{-3}), is

$$\begin{aligned} M_s &= \rho_s V_s & (1.7) \\ &= 7900 \text{ kg m}^{-3} \times 0.20 \text{ m}^3 \\ &= 1580 \text{ kg} \\ &\approx 1.6 \times 10^3 \text{ kg} \end{aligned}$$

Mathskit: Simple harmonic motion

1 The forces on the tube are the weight of the tube and the upthrust (buoyancy force) from the water. When the tube is in equilibrium these forces are balanced.

When the tube is pushed down an additional volume of water is displaced. The tube has a uniform cross section so the displaced volume is proportional to the vertical displacement. The upthrust is proportional to the volume of water displaced so, in this example, upthrust is proportional to the displacement.

The resultant force on the displaced tube is provided by the additional upthrust. This force is proportional to the displacement and in the opposite direction

When the tube is lifted above its equilibrium position, the volume of displaced water is reduced and upthrust due to the water is reduced by an amount that is proportional to the displacement. There is now a downward resultant force on the tube as the upthrust is less than the weight.

For both upward and downward displacements, the resultant force is proportional to the displacement and in the opposite direction, so the motion is SHM.

2

Step 1

The upthrust is equal to the weight of water displaced, and if the tube is a uniform cross section A , the upthrust is proportional to the length of tube below the water level. When the tube, mass m , is displaced a distance x an additional volume of water, V , is displaced.

$$V = Ax$$

The additional upthrust, F , is equal to the weight of the additional displaced water:

$$F = -Ax\rho g$$

where ρ is the density of water and g the gravitational field. F is directly proportional to x , and the negative sign indicates that the force is in the opposite direction to the displacement, so the motion is SHM.

Steps 2, 3, 4

$$F = ma = -Ax\rho g$$

so the acceleration, a , of the tube is

$$a = -(A\rho g/m)x$$

Therefore the constant relating acceleration to displacement is

$$\omega^2 = A\rho g/m$$

$$\omega = \sqrt{(A\rho g/m)}$$

(Skip Step 5 as the question does not refer to the amplitude)

Step 6

The period T of the oscillation is

$$\begin{aligned} T &= 2\pi/\omega \\ &= 2\pi \sqrt{(m/A\rho g)} \end{aligned}$$

3 An oil that is less dense than water would provide less buoyancy, so the test tube would float lower in the water, because the weight of oil displaced to equal the weight of the tube. For a displacement x the restoring force would be less, resulting in a smaller acceleration and longer time period. The equation for T in Question 2 shows that the time period is inversely proportional to the square root of the density ρ .

An oil that is more viscous would damp the motion, resulting in the amplitude of the oscillations decreasing more quickly.

At a glance: GPS

$$1 \quad v = \frac{2\pi R}{T} = \frac{2\pi \times 3 \times 10^7 \text{ m}}{5 \times 10^4 \text{ s}} = 3.8 \times 10^3 \text{ ms}^{-1}$$

$$2 \quad 24 \text{ hours} = 24 \times 60 \times 60 \text{ s} = 86400 \text{ s}$$

$$t_M = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{86400 \text{ s}}{\sqrt{1 - \frac{(3.8 \times 10^3 \text{ ms}^{-1})^2}{(3.00 \times 10^8 \text{ ms}^{-1})^2}}}$$

$$t_M = 86400.0000069 \text{ s}$$

So the clock appears to count an additional $6.9 \times 10^{-6} \text{ s}$, approximately $7 \mu\text{s}$

Note

In order to do this calculation as shown above, you need a calculator that displays a large number of places. If you are familiar with the mathematical technique of *binomial approximation*, you could use the following approach instead.

When $v^2 \ll c^2$,

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx 1 + \frac{1}{2} (v^2/c^2)$$

In this example

$$\begin{aligned} v/c &= 3.80 \times 10^3 \text{ m s}^{-1} / 3.00 \times 10^8 \text{ m s}^{-1} \\ &= 1.27 \times 10^{-5} \end{aligned}$$

$$v^2/c^2 = 1.60 \times 10^{-10}$$

This is very small so we can use the approximation. We can therefore write

$$t_M = t_0 \times \{1 + \frac{1}{2} (v^2/c^2)\}$$

and so

$$t_M - t_0 = t_0 \times \frac{1}{2} (v^2/c^2)$$

$$\begin{aligned} t_M - t_0 &= 86400 \text{ s} \times 1.60 \times 10^{-10} / 2 \\ &= 6.93 \times 10^{-6} \text{ s} \approx 7 \mu\text{s} \end{aligned}$$

Luis Alvarez: a versatile physicist

1 The curved paths suggest that the particles producing the paths are charged — charged particles take a curved path when moving at an angle to a magnetic field.

If two paths curve in opposite directions from a single point it suggests they were produced by particles with opposite charges.

If the path spirals inwards it suggests the particle is losing kinetic energy, slowing down.

If two paths appear spontaneously moving in opposite directions it suggests pair production from the decay of an uncharged particle, or from an interaction between uncharged particles.

2

a $10^9 \text{ eV} = 10^9 \times 1.6 \times 10^{-19} \text{ J} = 1.6 \times 10^{-10} \text{ J}$

b Use of the equation $E_k = \frac{1}{2} m v^2$ would give $v = 1.9 \times 10^{10} \text{ m s}^{-1}$, a speed greater than the speed of light ($c = 3.00 \times 10^8 \text{ m s}^{-1}$), which is not possible. For motion at speeds that approach the speed of light, equations of special relativity are needed. These equations show that, as v approaches c , a particle's mass appears to increase and its speed does *not* continue to increase as described by the classical equations; the speed gradually gets closer to c but can never quite equal c .

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